

MA 463 Abstract Algebra (3)

Semester / Term / Year: Fall 2024

Class Meeting Times / Location / Delivery Modality: TF, 12:35 to 1:50 PM / Miege 102 / In-Person

Seat Time: Our seat time for this 3-credit hour lecture course is three (3) hours per week. Even more, students are strongly encouraged to spend at least six (6) hours per week reviewing their course materials, practicing problems, working on assignments, and preparing for quizzes and examinations.

Instructor Information: Dr. Dylan C. Beck, Assistant Professor of Mathematics

Contact Information:

- Discord: <u>https://discord.gg/X3Nu6828mq</u>
- email: <u>Dylan.Beck@StMary.edu</u>
- Engage: <u>https://engage.stmary.edu/learn/course/view.php?id=8865</u>
- pronouns: he / him / his
- web page: <u>https://dylan-c-beck.github.io</u>

Office / Office Hours / Virtual Office: Miege 102A / MR, 9:00 to 11:30 AM / Microsoft Teams

University Academic and Classroom Policies:

Refer to Engage and the USM Catalog for the University Mission, Values, and Learning Objectives, ADA Academic Accommodations, and other University Academic and Classroom Policies. Particularly, students should become familiar with the following University policies. Course-specific policies are provided below.

- <u>Academic Catalog</u>
 - University Mission and Values Statements and Learning Outcomes
 - Academic Honesty Policy
 - University Attendance Policy
 - o Disability Services (or Academic Accommodations) Policy and Requesting Accommodations
- Academic Calendar
- Final Exam Schedule
- Student Handbook
 - o Disability and Accommodations Services
 - Student Code of Conduct
 - o Alcohol, Drugs, and Tobacco Policies
 - Campus Offices Information

Course Description

We will undertake an introduction to abstract algebra, covering elementary theoretical and computational aspects of number theory, groups, rings, and fields.

University Learning Outcomes

ULO 1.a Quantitative Literacy: The University of Saint Mary graduate will effectively access, evaluate, and apply information appropriately for a specific purpose.

Program Learning Outcomes

- 1. Develop problem solving and reasoning skills and analyze conceptual relationships. ULO 1.a
- 2. Apply disparate knowledge to solve complex problems in various disciplines and interpret the results in practical terms. ULO 1.a
- 3. Use mathematical abstraction and symbolism to create generalizations from observed patterns and develop specific examples from general statements. ULO 1.a
- 4. Read mathematical literature with increasing confidence, collaborate with others to solve problems, and effectively communicate mathematical concepts and processes. ULO 1.a
- 5. Appropriately use technological tools, such as graphing calculators and computers, as aids in solving problems, and correctly interpret the results that technology produces. ULO 1.a
- 6. Develop quantitative literacy by collecting, organizing and interpreting data, and create models for drawing trustworthy conclusions based on that data. ULO 1.a

Course Learning Outcomes

- 1. Explore the relationship between elements and operations. MPLO 1, 3
- 2. Examine the structures of groups, rings, fields, vector spaces, etc. MPLO 1, 3
- 3. Find cardinality and generators of groups. MPLO 1, 3
- 4. Calculate orbits and cycles. MPLO 1, 5
- 5. Evaluate homomorphisms between groups, rings, fields, etc. MPLO 1, 2, 3
- 6. Construct quotient groups, subgroups, ideals, cosets, etc. MPLO 1, 2, 3
- 7. Prove theorems about constructability of certain numbers. MPLO 1, 3
- 8. Decide which groups/fields/etc. are isomorphic. MPLO 1, 2, 5, 6

Course Materials

We will forego the use of a textbook for this course; rather, all material will be provided in the original lecture notes from Dr. Dylan C. Beck. Certainly, students may use other supplementary resources.

Course Policies

Class meetings will typically consist of an instructor-led lecture during which students will take notes, ask questions, and participate in discussion. Each student should be prepared to interact with the instructor, their classmates, and the course materials during each class meeting; the instructor may call on students to facilitate learning. Often, a short quiz with no more than ten questions pertaining to materials already introduced will be administered in class. Explicitly, on a quiz, students must be able to define key terms, answer true-false and multiple-choice questions, compute examples, and write short proofs. Exam reviews will be hosted during class prior to assessments. Consult the course schedule below for specific dates.

Each student must place their phone screen-side down on their desk at the beginning of each class period, and the device must be left in that position for the duration of the course meeting (barring extenuating circumstances that merit phone usage and must be communicated to the instructor). Each failure to comply with this policy will result in a deduction of one percentage point from the student's overall grade.

Regular and punctual attendance is vital to understanding the information presented in this course; however, in the event of an unavoidable absence, it is the responsibility of the student to inform the instructor by filling out the <u>Excused Absence Request Form</u> and to make arrangements with the instructor to make up any materials or assignments missed during class. Particularly, student-athletes should take care to maintain proactive and timely communication with the instructor to remain in good academic standing.

Even more, in view of the instructor's obligation to provide timely feedback on assignments, unless granted explicit permission from the instructor, students may not submit work more than 72 hours (or one class period) past the due date; toward this end, the instructor reserves the right to deny any student of comments or credit on any assignment submitted 72 hours (or one class period) past the due date or any assignment submitted after the rubric is posted. Crucially, unless granted explicit permission from the instructor, it is the responsibility of any student that misses class (due to athletics, illness, or other extenuating circumstances) to submit their own homework on time; it is strictly prohibited for students to submit work that is not their own. Exam make-ups must be completed within one week (or five business days) of the original exam date.

Conversely, the instructor reserves the right to deny any student comments or feedback on any assignment submitted in advance of the due date. Explicitly, the instructor may refuse to review, revise, or otherwise provide feedback on an assignment that has not been submitted to be graded by the instructor.

Communication between students and the instructor will occur primarily in the classroom and during the instructor's (virtual) office hours; however, each student should check their university email and <u>Engage</u> regularly for class announcements and supplementary materials. Even more, students are encouraged to join the <u>MA 463 Discord</u> for homework assistance and to discuss course materials, policies, and due dates with their classmates and the instructor. Crucially, engagement on Discord is optional: concretely,

- participation on Discord is not required as part of the MA 463 coursework;
- interactions on Discord will not influence a student's grade in the course; and
- Discord should not be viewed as an official medium of communication between the instructor and the students at the University of Saint Mary. Explicitly, Saint Mary email remains the official mode of communication between the students at the University of Saint Mary and their instructor.

Even more, students are forbidden from directly messaging the instructor via Discord, and any comments made by students on the MA 463 Discord may be viewed by their classmates. Bearing this in mind, confidential information must not be shared on Discord. Critically, the instructor is obligated to report any illegal, unethical, or improper activities on the Discord server to the appropriate authorities.

Generally, the instructor will adhere to the University of Saint Mary's <u>Hours of Operation</u> as it pertains to attending office hours, conducting make-up exams, and responding to emails or other correspondence from students; therefore, students should be conscientious and cognizant of this time frame when seeking assistance with assignments or scheduling appointments with the instructor. Even more, the instructor reserves the right to at least 48 hours (or two business days) to respond to any correspondence sent during the business week (Monday through Friday, excluding holidays) and more time on weekends.

Collaboration with classmates on homework is encouraged; however, each student is expected to submit their own work on all assignments, and each student will be graded on their work as it appears. Consequently, for students working together, it is critical that no party completes any work on behalf of another party and that each party determines their own solutions. Explicitly, students should write original solutions rather than copy from one another; however, students may discuss relevant techniques or strategies for their work. Ultimately, students must clearly indicate their collaborators for each assignment.

Outside of class, students should expect to spend at least two hours preparing materials and studying for every hour spent in class. Unlike in high school, students that do not understand coursework should not assume the instructor will repeat material until it is mastered; rather, each student is expected and encouraged to ask questions as they arise. Each student should certainly devote time to studying course materials outside of class, but in the event of confusion or difficulty, students should consider visiting the instructor during his office hours. **Do not hesitate to ask questions, as this course is cumulative.**

Crucially, it should be noted that the instructor reserves the right to refuse any accommodations requested by the student after the last day of classes of the semester, especially if these arrangements are negotiated to improve the student's course average or to achieve a desired letter grade in the class. Consequently, **it is imperative that students communicate early and often any difficulty or concern to the instructor**.

Course Prerequisites

Enrolled students must have a C or higher in MA 272 (Mathematical Reasoning) or instructor consent.

Course Assignment Points and Percentages

| type | quantity | weight | total |
|------------|----------|--------|-------|
| exam | 3 | 15% | 45% |
| final exam | 1 | 30% | 30% |
| homework | 10 | 1% | 10% |
| quiz | 15 | 1% | 15% |

Once during the semester, students can earn up to one percent toward their overall grade by completing the <u>MA 463 Syllabus Quiz</u>. Each student that completes all homework assignments with an average homework score of 67% may drop their lowest two homework scores. Each student that completes all quizzes with an average quiz score of 67% and passes the Review Quiz may drop their lowest three quiz scores.

Course Grade

| Α | 90% | 100% |
|---|-----|--------|
| В | 80% | 89.99% |
| С | 70% | 79.99% |
| D | 60% | 69.99% |
| F | 0% | 59.99% |

Our final examination will be administered on Tuesday, December 10 from 10:00 to 11:50 AM in Miege 102. Each of the three units will account for one-third of the exam. Completion of each of the three unit exams is required in addition to the final exam. Explicitly, any student that fails to complete any unit exam or the final exam fails the course. Even more, any student that fails the final exam automatically fails the course.

Coursework, Examinations, and Quizzes

Each week, at the instructor's discretion, relevant written homework will be issued to be completed outside of class. Consult the course schedule below for specific due dates regarding assignments. Late work may not be accepted unless proper documentation is provided. Even more, late work that is accepted may be subject to a deduction of one letter grade for each class period beyond the due date; however, if the student anticipates and communicates an issue to the instructor at least one calendar week prior to the assignment due date, the student may be allowed to submit their work after the due date with no deduction in points.

Every Friday of the semester, if an exam is not administered, a brief quiz will be administered in the last fifteen minutes of the class meeting. Unless otherwise specified, quiz questions will include definitions of key terms and formulas, true-false statements, multiple-choice questions, and computational and proofbased short response related to the material covered in class during an earlier course meeting.

Exams will be administered three times during the semester. Like with quizzes, students will use definitions and formulas to evaluate true-false and multiple-choice questions, compute examples, and write proofs. Credit for true-false and multiple-choice questions is awarded on an all-or-nothing basis. Credit for free response is earned by citing relevant definitions and theorems, demonstrating a command of appropriate techniques, and showing work: when the relevant work is provided and an answer is correct, full credit will be awarded. Partial credit may be awarded if some pertinent details are supplied; in this case, the instructor will typically mark the student's work with the phrase "work follows." Rubrics will typically be provided for the students' convenience. Each student must comply with the following exam regulations and guidelines.

- Each student is allowed a writing utensil and a non-graphing, non-programmable scientific calculator without computer algebra capabilities during exams. Graphing calculators are prohibited.
- Other papers and electronic devices must be stored in the student's backpack and placed under or next to their desk. Cell phones, smart watches, notes, and textbooks are prohibited during exams.

Once all students have cleared their workspaces, the assessment will begin. Each student will then have the full duration of the scheduled course meeting to complete the examination. Once the student has finished the assessment (or time has expired), the check-out procedure must be initiated by the student submitting their work to the instructor; the student may then be dismissed with the instructor's permission.

Below is a brief list of authorized handheld calculators for use on quizzes and exams.

| TI-30XIIS | TI-34 |
|-----------|--------------------|
| TI-30XA | Casio fx-300ESPLUS |

Calculators other than those listed above may be used with the instructor's explicit permission.

Update Clause

Ultimately, the instructor reserves the right at any point in the semester to alter this syllabus to reflect changes in policy or schedule due to extenuating or unforeseen circumstances. Consequently, it is the responsibility of the students to remain up to date with this syllabus; however, the instructor will inform students of any such changes to this document, and the syllabus will be maintained and updated.

Course Schedule

Week 1: Review of Logic, Proofs, and Sets

Friday, August 30 — Last Day to Add a Course

Friday, August 30 — Quiz 1: Review of Sets, Functions, and Relations

| date | section | topic(s) |
|--------------------|---|--|
| Tuesday, August 27 | Chapter 0: Essential Topics in Logic, Proofs, and Sets | proof techniques properties of functions properties of relations set operations |
| Friday, August 30 | Chapter 0: Essential Topics in Logic, Proofs, and Sets | congruence modulo <i>n</i> Division Algorithm Mathematical Induction |

Week 2: Cyclic Groups

Monday, September 2 — Labor Day Holiday Friday, September 6 — Last Day to Drop a Course with 100% Refund Friday, September 6 — Quiz 2: Basic Definitions, Examples, and Properties of Groups

| date | section | topic(s) |
|----------------------|--|--|
| Tuesday, September 3 | 1.1: Group Definitions, Examples 1.2: Group Properties, Subgroups | associativity binary operations multiplicative identity multiplicative inverse subgroups Subgroup Tests |
| Friday, September 6 | 1.3: Cyclic Groups | cyclic group generators order (of a group or element) properties of cyclic groups |

Week 3: Complex Numbers, Dihedral Groups, and the Symmetric Group

Monday, September 9 — First Day of Withdraw Period Tuesday, September 10 — Quiz 3: Cyclic Groups Wednesday, September 11 — Fall 2024 Convocation Friday, September 13 — Quiz 4: Complex Numbers as a Group Under Multiplication

| date | section | topic(s) |
|-----------------------|--|--|
| Tuesday, September 10 | 1.4: Complex Numbers | circle group complex conjugation De Moivre's Theorem roots of unity |
| Friday, September 13 | 1.5: the Symmetric Group 1.6: Dihedral Groups | cycles length (of a cycle) permutations rigid motions transpositions |

Week 4: Cosets, Lagrange's Theorem, and Quotient Groups

Tuesday, September 17 — Quiz 5: the Symmetric and Dihedral Groups Friday, September 20 — Quiz 6: Cosets and Lagrange's Theorem

| date | section | topic(s) |
|-----------------------|-------------------------|--|
| Tuesday, September 17 | 1.7: Lagrange's Theorem | cosetsLagrange's Theorem |
| Friday, September 20 | 1.8: Quotient Groups | normal subgroupsquotient groups |

Week 5: Group Homomorphisms and Isomorphisms

Monday, September 23 — Last Day to Drop a Course with 50% Refund Tuesday, September 24 — Quiz 7: Normal Subgroups and Quotient Groups

| date | section | topic(s) |
|-----------------------|--------------------------|---|
| Tuesday, September 24 | 1.9: Group Homomorphisms | conjugationgroup homomorphismgroup isomorphismkernel |
| Friday, September 27 | 1.10: Group Isomorphisms | Group Isomorphism Theorems |

Week 6: Chapter 1 Review and Exam 1

Tuesday, October 1 — Quiz 8: Group Homomorphisms and Isomorphisms

| date | section | topic(s) |
|--------------------|------------------|----------|
| Tuesday, October 1 | Chapter 1 Review | |
| Friday, October 4 | Exam 1 | |

Week 7: Ideals, Quotients, Rings, and Ring Homomorphisms

Monday, October 7 — Graduation Application Deadline for Spring or Summer 2025 Graduates Friday, October 11 — Quiz 9: Rings and Ring Homomorphisms

| date | section | topic(s) |
|--------------------|--------------------------------------|---|
| Tuesday, October 8 | 2.1: Rings and Ring Homomorphisms | characteristicmultiplicative inversesring homomorphisms |
| Friday, October 11 | 2.2: Ideals and Quotient Rings | generators ideals quotient rings subrings |

Week 8: Ring Isomorphism Theorems

Tuesday, October 15 — Quiz 10: Ideals and Quotient Rings Thursday, October 17 to Sunday, October 20 — Fall Break

| date | section | topic(s) |
|---------------------|--------------------------------|---|
| Tuesday, October 15 | 2.3: Ring Isomorphism Theorems | evaluation homomorphism Ring Isomorphism Theorems |

Week 9: Field, Integral Domains, Maximal, and Prime Ideals

Monday, October 21 — Spring Priority Registration Opens for Current Juniors and Seniors Tuesday, October 22 — Quiz 11: Ring Isomorphisms Theorems Friday, October 25 — Quiz 12: Integral Domains and Fields

| date | section | topic(s) |
|---------------------|----------------------------------|---|
| Tuesday, October 22 | 2.4: Integral Domains and Fields | (finite) fields integral domains zero divisors |
| Friday, October 25 | 2.5: Prime and Maximal Ideals | existence of maximal ideals maximal ideals prime ideals quotient rings |

Week 10: Chapter 2 Review and Exam 2

Monday, October 28 — Last Day to Withdraw and Receive a Grade of W Monday, October 28 — Spring Priority Registration Opens for Current Freshmen and Sophomores Tuesday, October 29 — Quiz 13: Prime and Maximal Ideals

| date | section | topic(s) |
|---------------------|------------------|----------|
| Tuesday, October 29 | Chapter 2 Review | |
| Friday, November 1 | Exam 2 | |

Week 11: Polynomial Irreducibility and Polynomial Rings

Monday, November 4 — Spring Registration Opens for New Registering Students

| date | section | topic(s) |
|---------------------|------------------------------------|--|
| Tuesday, November 5 | 3.1: Polynomial Rings and Division | Factor TheoremPolynomial Division AlgorithmRemainder Theorem |
| Friday, November 8 | 3.2: Polynomial Irreducibility | Eisenstein's Criterion Gauss's (Little) Lemma polynomial irreducibility reduction modulo <i>p</i> |

Week 12: Field Extensions and Simple Field Extensions

Tuesday, November 12 — Quiz 14: Polynomial Irreducibility

| date | section | topic(s) |
|----------------------|------------------------|--|
| Tuesday, November 12 | 3.3: Field Extensions | Fund'l Theorem of Field Theory |
| Friday, November 15 | 3.4: Simple Extensions | algebraic extensionsminimal polynomials |

Week 13: Finite Field Extensions and Chapter 3 Review

Tuesday, November 19 — Quiz 15: Roots of Polynomials and Field Extensions

| date | section | topic(s) |
|----------------------|------------------------|---|
| Tuesday, November 19 | 3.5: Finite Extensions | degrees of finite field extensions Fundamental Theorem of FGFE towers of field extensions |
| Friday, November 22 | Chapter 3 Review | |

Week 14: Exam 3

Wednesday, November 27 to Sunday, December 1 — Thanksgiving Break

| date | section | topic(s) |
|----------------------|---------|----------|
| Tuesday, November 26 | Exam 3 | |

Week 15: Final Exam Review

Monday, December 2 - Last Day to Withdraw and Receive a Grade of WP or WF

| date | section | topic(s) |
|---------------------|-------------------|----------|
| Tuesday, December 3 | Final Exam Review | |
| Friday, December 6 | Final Exam Review | |

Week 16: Finals Week

Tuesday, December 10 — MA 463 Final Examination (10:00 to 11:50 AM), Miege 102 Thursday, December 12 — Last Day of Fall 2024 Semester Friday, December 13 — Final Grades Due to Registrar by 5:00 PM Saturday, December 14 — Fall 2024 Commencement

| date | section | topic(s) |
|----------------------|-------------------|----------|
| Tuesday, December 10 | Final Examination | |